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"Do Nows" Done Better: Ten Strategies for Fortifying the First Five Minutes of Every Mathematics Class (Part 1of 3)

by Robert Gerver, North Shore High School, rgerver@optonline.net
and Richard Sgroi, Fox Lane High School, dr.rsgroi@gmail.com

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by Robert Gerver, North Shore High School, rgerver@optonline.net
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"Do Nows" Done Better: Ten Strategies for Fortifying the First Five Minutes of Every Mathematics Class (Part 3 of 3)

by Robert Gerver, North Shore High School, rgerver@optonline.net
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"Do Nows" Done Better: Ten Strategies for Fortifying the First Five Minutes of Every Mathematics Class (Part 1 of 3)

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In all presentations, "getting off to a good start" is essential. This is particularly important in teaching mathematics. The way that a lesson begins is critical to the way it will evolve, flow, and reach closure. Lessons that begin with purpose and direction set both a constructive tone that adds to the learning environment and an instructional routine with which students become familiar and comfortable. Being cognizant of this and knowing how to execute the perfect lesson opener are two completely different things. In our years as teachers of mathematics, we developed and refined different strategies to start lessons. These openers *jump start* the lesson in a variety of ways.

Looking back on the many openers we have created and used over the years, in courses from basic mathematics to Advanced Placement courses, we have been able to sort these "Do Nows" into ten distinct categories:

1. **Find and Fix**
2. **What's the Problem?**
3. **Notebook Scavenger Hunt**
4. **Using Quotations**
5. **Standardized Test Questions**
6. **Lesson Readiness**
7. **Lesson Explorations**
8. **Using Non-English Math Textbook Problems**
9. **Partner Problems**
10. **Related Recreational Activities**

This is the first of 3 articles on this topic. What follows is a look at the first five of the categories, offering sample jump start activities as well as suggestions to tailor activities to meet your own personal style and that of your students. The second five activities will be presented in a future article.

1. Find and Fix

In this type of Do Now, students are given a problem with a detailed solution. The solution contains one or more errors and the student's job is to find and fix the error(s). For an example, see the *Find and Fix* problem below:

> **Find and Fix**
>
> **Subtract $2x - 7$ from $x^2 + 4x + 6$**
>
> ***Find and fix this solution:***
>
> $x^2 + 4x + 6 - 2x - 7 =$
>
> $x^2 + 4x - 2x + 6 - 7 =$
>
> $x^2 + (4x - 2x) + (6 - 7) =$
>
> $x^2 + 2x + (-1) =$
>
> $x^2 + 2x - 1$

In this example, students must recognize that the entire binomial $(2x - 7)$ needs to be subtracted from the trinomial and a set of parentheses around $2x - 7$ is needed to correctly complete the solution. The solution must begin with $x^2 + 4x + 6 - (2x - 7)$, which leads to $x^2 + 4x + 6 - 2x + 7$. Once students have identified the error, they must follow the proper processes to fix it, determine the correct answer and justify their corrections.

2. What's the Problem?

This Do Now focuses on problem posing. Here, students are given a solution to a verbal problem in the form of a numerical equation. They are then asked to create a verbal problem for which the given equation would be the solution. See the sample *What's the Problem* solution below:

> **What's the Problem?**
>
> ***Examine the equation below used for exponential depreciation calculations. Look through this chapter and your notes to help you write a problem that could be modeled by this equation:***
>
> $$19{,}777.84 = 24{,}800(1 - 0.055)^4$$

This Do Now was assigned after students learned the exponential decay formula $y = a(1 - r)^x$ where y is the current amount, a is the initial amount, r is the decay rate expressed as a decimal, and x is the time under consideration. One possible scenario for this sample solution is:

A car is originally priced at $24,800. It depreciates at a rate of 5.5% per year. What is the car worth after 4 years?

3. Notebook Scavenger Hunt

This Do Now works well as part of a year-long initiative designed to have students take notes and keep the notes organized. In Notebook Scavenger Hunt, students are given quests to retrieve information related to the current topic being studied. This requires them to have their notes for that unit. Knowing that they will have to comb through their notes, students have an incentive to take better notes during every class and organize them well. Here are some examples of *Notebook Scavenger Hunt* questions:

> **Notebook Scavenger Hunt**
>
> ✔ **On March 8, we found the perpendicular to the tangent line to a circle at a point P. What was the center of the circle?**
>
> ✔ **On November 7, we computed the monthly mortgage payment for a family's home purchase. What was the family's last name?**
>
> ✔ **How many words were there in the essential question for our lesson on April 4?**

Notebook Scavenger Hunt questions can even appear as bonuses on a test. Encouraging students to take and keep good notes fosters better focus in class, and provides the students with more support when they have questions.

4. Standardized Test Familiarity

Some mathematical exams have problems with more convoluted instructions than the straight multiple-choice question. It is important that the student can concentrate on the mathematics, and not be confounded or intimidated by the layout of the question. Problems that follow the style of the AP, SAT, or State test questions can be given in small frequent doses, so students get used to the types of questions they will see. Students who have months of experience tackling these nontraditional instructions treat them as "second nature" since they have seen them frequently. The result of this type of practice allows the student to concentrate on the mathematics while feeling confident with the format. Additionally, due to the increased profile of high-stakes testing, most mathematics teachers like to spiral material, and this is a great way to accomplish that strategy.

Continued on page 18 ☞☞

Examine the two examples given below:

Standardized Test Question

Look at expressions I and II below. Then choose the statement which is true for all real values of x.

I: $|7 + x|$ II: $|7 - x|$

a) Expression I is always greater than Expression II.

b) Expression II is always greater than Expression I.

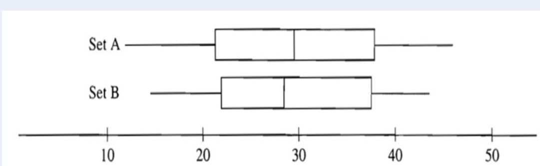
c) Expressions I and II can never be the same value.

d) Expressions I and II are always equivalent.

e) Without knowing the value of *x*, it is not possible to determine which expression is greater.

Standardized Test Question

The side-by-side box plots below summarize two data sets, A and B.



I. Set A contains more data than Set B.

II. The median of Set A is greater than the median of Set B.

III. The data in Set A have a larger range that the data in Set B.

Which of the following statements about the three sentences is true?

a) I only
b) I and II only
c) III only
d) II and III only
e) I, II, and III

5. Quotations

Students often remark how they love debates and lively class discussion in humanities classes. There are an unlimited number of quotes available online that can be used to jump start a discussion in a mathematics class. Everyone gets a chance to voice their interpretation. It is enjoyable when students start off a mathematics class with a little levity, philosophy, and opinion! Here are several examples:

Quotations

- ✔ *Never try to walk across a river just because it has an average depth of four feet.* **-Milton Friedman**
- ✔ *If equations are trains threading the landscape of numbers, then no train stops at pi.* **-Richard Preston**
- ✔ *Do not worry too much about your difficulties in mathematics, I can assure you that mine are still greater.* **-Albert Einstein**
- ✔ *We must admit with humility that, while number is purely a product of our minds, space has a reality outside our minds.* **-Karl Friedrich Gauss**
- ✔ *Obvious is the most dangerous word in mathematics.* **-E.T. Bell**

It's Up to You!

All of these Do Now activities can be used to test prerequisite skills, spiral skills learned in past lessons, or to review for quizzes and exams. How you choose to jump start your lesson is up to you. Pick your favorites and use them in whatever frequency you decide. There is unlimited flexibility within each of the options we have presented. We advise you to team up with teachers teaching the same course to create a bank of shared Do Now activities.

If you feel that these new beginnings can make your lessons more productive and enjoyable, look for our article on five more Do Now activities in an upcoming issue of the *CMC Communicator*.

"Do Nows" Done Better: Ten Strategies for Fortifying the First Five Minutes of Every Mathematics Class (Part 2 of 3)

by Robert Gerver, North Shore High School, rgerver@optonline.net
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Welcome back to our discussion on period-opening activities that engage students in all mathematics classes. This is a continuation of our article which appeared in the December 2020 issue of *CMC ComMuniCator*. The premise of both of these articles is that successful mathematics lessons on all ability levels need productive and purposeful beginnings. The way that a lesson begins is critical to the way it will evolve, flow, and reach closure. In the first article, we addressed these five "*Do Nows" Done Better:*

1. ***Find and Fix***
2. ***What's the Problem?***
3. ***Notebook Scavenger Hunt***
4. ***Using Quotations***
5. ***Standardized Test Questions***

In this article, we will continue with the next three more beginning of class strategies:

6. ***Lesson Readiness***
7. ***Lesson Explorations***
8. ***Using Non-English Math Textbook Problems***

In our final article, we will discuss the last two period-opening strategies:

9. ***Partner Problems***
10. ***Related Recreational Activities***

The full set of ten strategies give the teacher a powerful portfolio of pedagogical tools.

6. Lesson Readiness

Some lessons require some tedious work to get started. This is particularly relevant with certain statistics and manipulative lessons. Many lessons require students to enter lists of data into electronic devices. This data can be posted on the board, found by students online, or handed out on paper. Data entry starts as soon as the students enter the room so they are ready to do pure mathematics by the time the bell rings. This procedure is very helpful in AP Statistics classes and other data driven lessons.

When manipulative activities require students to prepare materials in advance, students start on the task as soon as they enter the room. Here is an example of deriving the area formula for a trapezoid (***Figure 1***).

Take scissors and cut the two index cards to form two congruent trapezoids. Label the height of each trapezoid *h*, and the bases b_1 and b_2 as shown below.

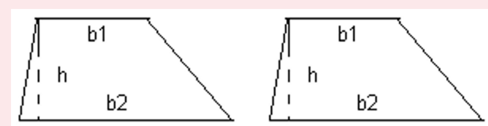

Figure 1: Lesson Readiness—Transformation of Trapezoids

After students cut and label each trapezoid, they are ready to begin the lesson. Direct the students to rotate the second trapezoid and place it against the first trapezoid to form a parallelogram (***Figure 2***). Students know the area of a parallelogram formula from their geometry class. Students use the formula $A = bh$ to get the area of the large parallelogram formed by the two trapezoids.

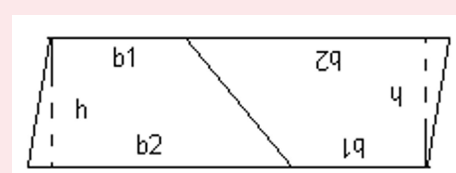

Figure 2: Lesson Readiness—Transformation of Trapezoids

The area of the parallelogram is:

$$A = h\,(b_1 + b_2)$$

Therefore, the area of one trapezoid is:

$$A = \tfrac{1}{2}\,h\,(b_1 + b_2)$$

During the Do Now, the transformation and the derivation of the formula can be part of the lesson, or you can use guiding questions to help them uncover the formula themselves. Since students cut out and label the trapezoids as soon as they walk in the room, there is more time in the lesson for the mathematics.

7. Lesson Explorations

As compared to the Lesson Readiness Do Now number 6 , Lesson Explorations are content-centered. Begin with mathematical exploration activities that set the stage for the lesson to follow such as summarizing data, generating data, analyzing graphs, making conjectures, exploring patterns and relationships, etc. Assign the sample below (***Figure 3***) at the beginning of a geometry lesson on the converse of the Pythagorean theorem.

Compare the sum of the squares of the two smaller sides of these triangles to the square of the largest side and express them as an inequality. Once completed, classify the triangles as obtuse, right, or acute.

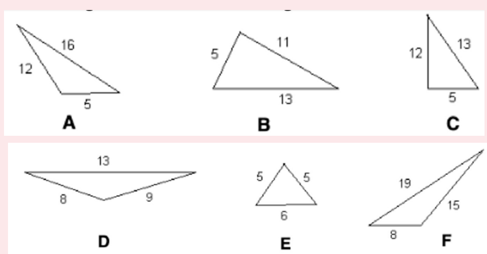

Figure 3: Lesson Exploration–Pythagorean Theorem

When the lesson begins, students are asked to form a conjecture about what they found. Notice that the generation of the information necessary to make the conjecture is started as soon as the students enter the room. Students start thinking mathematically, looking for patterns, and making conjectures before the lesson begins.

8. Using Non-English Math Problems

Problems written and solved in languages other than English (***Figure 4***) can be found by searching the internet or using Google Translate. Challenge students to interpret part or all of it, or even solve the problem! Their familiarity with the mathematics presented will serve as cues for a culturally rich, enlightening Do Now activity. Select a few words from the non-English language text and ask students to translate them based upon their familiarity of the mathematics. More often than not, the mathematics enriches the written words to the point they are able to translate.

Examine this math problem written in French. What looks familiar to you? What do you think this problem is about? What method is being described here? What might the translation of the French word "inconnue" be?

SYSTEME DE DEUX EQUATIONS A DEUX INCONNUES

Elimination d'une inconnue par addition

(1) $4x + 2y = 2$
(2) $6x - 8y = 58$

Multiplions par 4 les deux members de (1)

(3) $16x + 8y = 8$

Ajoutons (2) et (3):

(4)
$$16x + 8y = 8$$
$$6x - 8y = 58$$
$$22x = 66$$
$$22x/22 = 66/22$$
$$x = 3$$

Portons x = 3 dans (1):
$$4(3) + 2y = 2$$
$$12 + 2y = 2$$
$$12 + 2y - 12 = 2 - 12$$
$$2y = -10$$
$$2y/2 = -10/2$$
$$y = -5$$

Portons y = –5 dans (2):
$$6x + 40 = 58$$
$$6x + 40 - 40 = 58 - 40$$
$$6x = 18$$
$$6x/6 = 18/6$$
$$x = 3$$

La solution est (3, –5)

Figure 4: Non-English Problem

Students should recognize that this is a system of two linear equations with two unknowns (inconnues). Students are often intrigued by the fact that even cultures with different alphabets still do their algebra using good old *x* and *y*! Even in languages such as Hebrew, in which reading is done right to left, graphs read from left to right!

Make a Difference!

Do Nows are field-tested strategies that offer many mathematical dividends. They can be used to test current unit skills, prerequisite skills, and spiral skills learned in past lessons. We encourage you to team up with other teachers to create a bank of shared Do Now activities. Look for our Do Now article in an upcoming issue of the CMC *ComMuniCator*.

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1. ***Find and Fix***
2. ***What's the Problem?***
3. ***Notebook Scavenger Hunt***
4. ***Using Quotations***
5. ***Standardized Test Questions***
6. ***Lesson Readiness***
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In this final article, we discuss the last two period-opening strategies:

9. ***Partner Problems***
10. ***Related Recreational Activities***

The full set of ten strategies give the teacher a powerful portfolio of pedagogical tools.

9. Partner Problems

In this "Do Now," students first work individually and then collaboratively on a paired set of problems (***Figure 1***).

A. What is the y-intercept of this linear equation? $-2(2x - 3y) = 18$

B. Find the median of the following set of data:
18, 2, 2, 4, 2, 1, 4, 6, 2, 8

Figure 1: Partner Problem

Prior to the activity, assign each student a partner in any way that works for you. Some teachers assign partner A and B the day before; others assign partners as students walk in the classroom by telling students they are an A or B in a specific group. Still others have a deck of index cards (labeled A1, B1, A2, B2, A3, B3, etc.) sitting at the door for students to pick up as they walk in.

Here is how this "Do Now" works:

- ✔ Develop a set of two disparate problems that have exactly the same numerical answer. Label one A and the other B.
- ✔ Students work in pairs. Assign one as partner A and the other as partner B.
- ✔ Distribute the labeled problems to the corresponding partner.
- ✔ Students work on their assigned problem on their own.
- ✔ Once completed, they share answers. If the answers to the different problems are the same, the work is correct. If the answers are different, both students work together to find the correct solution.

Partner solutions must be the same. If they are not, either one or both of the partners are incorrect. They should work together to find the correct solution. In ***Figure 1***, the y-intercept in problem A is 3 and the median in problem B is 3 as well. You might recognize that the element of surprise when both answers are the same is lost after the first time the activity is done. While the "Aha! moment" may be gone, as you continue to assign partner "Do Nows," students will know that their answers must match and will work to make sure that they do.

10. Related Recreational Activities

These short activities use puzzles, optical illusions, and classic/historical problems in mathematics that directly relate to the content being studied. For example, during the unit on properties of parallelograms, the teacher can present the following optical illusion (***Figure 2***, page 27).

Which segment is longer, AB or BC?

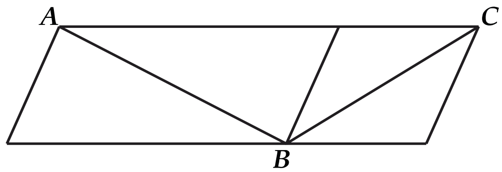

Figure 2: Optical Illusion Example

Students explain the illusion and offer possible reasons, using the geometry they know. The deception lies in the fact that segment *AB* is the shorter diagonal of a wider parallelogram, and segment *BC* is the longer diagonal of a smaller parallelogram.

Look at the following illusion (***Figure 3***) which is used during the geometry unit on intercepted arcs.

If the following arcs were completed, which of the following circles—1, 2, or 3—would have the greatest area?

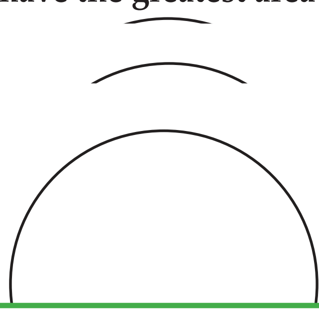

Figure 3: Intercepted Arcs

Students usually guess arc 1, however, all three arcs are from the same circle! Students love optical illusions, and they are readily available on the Internet. Search for them and align them to specific units, and not as just arbitrarily chosen, drop-in activities.

Classic/historical problems are great recreational activities. You can find classic and famous historical problems by doing an Internet search. The following two examples (***Figure 4*** and ***Figure 5***) can be used as "Do Nows" in high schools and middle schools. They are unsolved problems in mathematics that can be explained to an elementary school student!

In an addition problem, the numbers that are being added are called **addends**. Pick any even integer greater than 2. Express it as the sum of two prime addends.

Figure 4: Goldbach's Conjecture

The problem in ***Figure 4*** is known as Goldbach's Conjecture and is an example of an unsolved problem in mathematics: Can every even integer greater than 2 be expressed as the sum of two prime addends? Let your students try more even numbers for a few minutes.

Here is another unsolved problem:

A **prime number generator** is a formula that outputs only prime numbers for all integers that are inputed.

Consider the function $f(x) = x^2 - x + 41$.

Substitute the numbers 0–40 for x. Check your answers with an Internet list of prime numbers.

- Are all of your answers prime?
- Is $f(x) = x^2 - x + 41$ a prime number generator?

Figure 5: Unsolved Problem—Prime Number Generator

In ***Figure 5***, the expression $x^2 - x + 41$ does, in fact, yield primes for the first 41 trials (0–40). However, when $x = 41$, the result is not prime. Students are amazed that a formula works 41 consecutive times and then fails! (It is a good example of why we need proofs in mathematics.) It is unknown whether or not a prime number generator exists.

The oldest unsolved mathematics problem is can you find an odd perfect number problem (**bit.ly/OddPerfectNumber**). Can you find an odd perfect number?

These activities offer many mathematical dividends. Students often do not realize that there *are* unsolved problems in mathematics, and some of them can be explained to a fourth grader! Additionally, even students who *do* realize that there are unsolved problems often think that the unsolved problems must be of some super-advanced level way beyond graduate level mathematics.

Make a Difference!

Do Nows are field-tested strategies that offer many mathematical dividends. They can be used to test current unit skills, prerequisite skills, and spiral skills learned in past lessons. We encourage you to team up with other teachers to create a bank of shared Do Now activities, and then share them with the CMC *ComMuniCator* for possible future publication.